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TURBULENT HEAT TRANSFER FROM THE CORE TUBE IN THERMAL ENTRANCE REGIONS OF CONCENTRIC ANNULI

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Abstract—The problems of thermal boundary-layer growth and heat transfer, for hydrodynamically fully developed turbulent flow in concentric annuli, are investigated by means of the momentum and heat-transfer integral equations, along with a modified universal velocity profile.

The investigation was conducted for a range of radius ratios from 1·01 to 5, Prandtl numbers from 0·01 to 30 and a Reynolds number range of from 10000 to 200000.

The results reveal that in general the heat-transfer coefficient attains the fully developed value in less than thirty equivalent diameters; also that the entrance length is moderately dependent upon radius ratio.

There is good agreement between the present analysis and existing experimental data for the fully developed temperature profile regions.

NOMENCLATURE

a, thermal diffusivity $(k/c\rho)$;
c, specific heat at constant pressure;
De, equivalent diameter of annulus, $2(r_2 - r_1)$;
h, convective heat-transfer coefficient;
k, thermal conductivity;
q, heat flux;
r, radius;
T, temperature;
u, velocity in x direction;
Y, modified wall distance for annulus, equation (6);
x, distance in flow direction from beginning of heated section;
r^+, distance parameter $(r\,u_2^*/\nu)$;
u_2^*, friction velocity $(\tau_2/\rho)^{0\cdot5}$;
T^+, dimensionless temperature parameter,
$$\frac{(T_1 - T)\,c\tau_2}{q_1\,u_2^*};$$
U^+, modified velocity parameter, concentric annulus (u/u_2^*);
Y^+, modified distance parameter, concentric annulus $(Y\,u_2^*/\nu)$;
Nu, Nusselt number;
Pr, Prandtl number;
Re, Reynolds number.

Greek symbols
α, radius ratio (r_2/r_1);
δ_h, thickness of thermal boundary layer;
ε, eddy diffusivity;
μ, absolute viscosity;
ν, kinematic viscosity;
ρ, density;
σ, eddy diffusivity ratio $(\varepsilon_H/\varepsilon_M)$;
τ, shear stress;
ϕ_1, ϕ_2, functions;
δ_h^+, non-dimensional thermal boundary-layer thickness, $\delta_h/(r_2 - r_1)$.

Subscripts
1, inner wall or inner region of annulus;
2, outer wall or outer region of annulus;
b, bulk;

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d, fully developed;
H, heat;
h, edge of thermal boundary layer;
i, inlet;
M, momentum;
m maximum;
x, local.

1. INTRODUCTION

THERMAL entrance effects are always present, when a fluid flowing adiabatically into a passage, enters a region having a wall temperature different from that of the fluid. On entering the heated section, a new temperature distribution will be set up within the fluid; this may take various forms depending upon the heat flux boundary conditions and past history of the fluid. At the cross section where heating commences the temperature gradient in the fluid near the wall is theoretically infinite; the resulting local heat-transfer coefficient is, in theory, likewise infinite and decreases in the direction of fluid flow until a constant value is reached if constant fluid properties are assumed.

Frequently the flow conditions or the design of a heat-transfer section are such that the thermal entrance region effects are significant and cognizance of the thermal entry length must be taken. In other cases, with very long flow passages, the entry length is only a small section of the whole length, and can be often ignored.

In recent years, a considerable amount of work has been carried out on the problem of fully developed turbulent flow and heat transfer in annuli, both concentric and eccentric. The problem is of some practical importance as these configurations are used in nuclear reactors and in some types of heat exchanger. A great deal of work has been done on the problem of turbulent flow and heat transfer in the entrance regions of tubes and between parallel plates. Only limited information is available on the growth of turbulent thermal boundary layer in an annulus [1].

The only experiments regarding to the hydrodynamic entrance region in annuli appear to be those of Rothfus *et al.* [2] and Olson and Sparrow [3].

Results of Rothfus *et al.* on the entrance length of concentric annuli indicate a behaviour which is quite different from the results for tubes and parallel plate flow passages. As pointed out by Olson and Sparrow [3], the entrance lengths from the annulus data of Rothfus *et al.* [2] appear to be larger by a factor of ten, than the typical tube entrance lengths (for instance [4, 5, 6]) and to be strongly affected by the Reynolds number.

To explore this apparent anomaly, Olson and Sparrow investigated the entrance region by measuring the wall static pressures, for water flow in annuli (radius ratios of 2 and 3·2) and in a circular tube, fitted with interchangeable square or rounded entrances. The test covered the Reynolds number range from 16000 to 70000. For the annuli, Olson and Sparrow found that the length required to approach to within 5 per cent of the fully developed pressure gradient was about twenty to twenty-five hydraulic diameters. This is in general accord with entrance length results for tube and parallel plates but differs from those of Rothfus *et al.* [2].

If the velocity profile of the fluid entering a heat-transfer passage is not developed, the entry effect will differ from that where the velocity profile is fully developed at the entrance. The case of purely thermal entrance effect in concentric annuli when the fluid flow is already fully developed at the point where heating commences, may be dealt with theoretically using semi-empirical correlations for the velocity and heat diffusion properties of the flow.

Using a boundary-layer model and integral methods, a similar approach to that used by Deissler [4] for tubes, is herein applied to the case of thermal entrance region heat transfer from the core of a concentric annulus; the case of heat transfer from the core being the one most usually encountered in practice. It is also assumed here that the fluid enters the annulus with a uniform temperature and fully developed turbulent velocity profile.

2. ANALYSIS

2.1. *Energy equation*

The energy equation for the thermal boundary layer in a concentric annulus can be written from the diagram, described by Fig. 1 in the idealized model, as

$$[\int_{r_1}^{r_h} T\rho uc 2\pi r\, dr]_1 + 2\pi r_1\, dx\, q_1 + \{[\int_{r_1}^{r_h} \rho uc 2\pi r\, dr]_2 - [\int_{r_1}^{r_h} \rho uc 2\pi r\, dr]_1\} T_h = [\int_{r_h}^{r_h} T\rho uc 2\pi r\, dr]_2 \qquad (1)$$

where 1 and 2 refer to the planes indicated. For small heat fluxes, the properties of the fluid may be assumed to be constant and for uniform heat flux at the wall of the core tube, an integration of equation (1) then reduces, in terms of dimensionless parameters defined in the Nomenclature, to

$$\frac{X}{De} = \frac{1}{2r_1^{+2}(\alpha - 1)} \int_{r_1^+}^{r_h^+} (T_h^+ - T^+)\, U^+ r^+\, dr^+. \qquad (2)$$

The length x for a given thermal boundary-layer thickness δ_h can be calculated from equation (2) provided that U^+ and T^+ are known functions of r^+. It is assured that $\delta_h = 0$ at $x = 0$.

In the present study, the velocity profile at the commencement of the heated section is that for fully developed turbulent flow, whereas the temperature is uniform, T_i at $x = 0$.

2.2. *Temperature distribution*

In order to obtain the temperature distribution, use is made of the concept of eddy diffusivity, ε, and the basic equations governing the transport of momentum and heat can thus be written as:

$$\frac{\tau}{\rho} = (\nu + \varepsilon_M)\frac{\partial u}{\partial r} \qquad (3)$$

and

$$-\frac{q}{c\rho} = (a + \varepsilon_H)\frac{\partial T}{\partial r}. \qquad (4)$$

The temperature distribution $T(r)$, for a given wall heat flux and fluid can be determined from equation (4) if $\varepsilon_H(r)$ and $q(r)$ are both known. Before undertaking the solution of this equation it is necessary to know the relation $u(r)$ for given values of α, and also to know the variation of $(\varepsilon_H/\varepsilon_M)(r) = \sigma(r)$ where $\varepsilon_M(r)$ can be evaluated from equation (3).

The evaluation of $\varepsilon_M(r)$ requires a knowledge of $\tau(r_1)$ and $u(r)$. The shear stress and velocity distribution in fully developed pipe flow are well defined, but in the annulus geometry there is some uncertainty regarding the positions of radius of maximum velocity and zero shear [7].

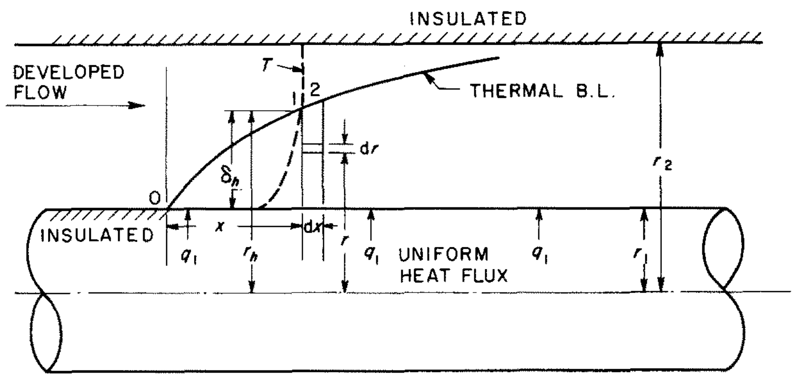

FIG. 1. Idealized model.

Although the positions of radius of maximum velocity is definitely less than for laminar flow with the deviation being greater for larger radius ratios [1, 7, 8], in the present analysis it is assumed that zero shear stress occurs at the radius of maximum velocity and that the radius of maximum velocity in annular turbulent flow coincides with Lamb's [9] radius of maximum velocity. The latter assumption is reasonable in the range of radius ratios smaller than five in the light of the findings of [1, 8, 11]. The displacement of the radius of maximum velocity in fully developed flow due to heat transfer is also assumed negligible.

It is well-known fact that the standard universal velocity distribution is not fully adequate for the inner velocity profile of a concentric annulus [7, 8, 10]. For the velocity distribution, $u(r)$, a *modified* universal velocity profile is used. The experimental velocity distributions in concentric annuli reported in [7, 12, 13], are closely correlated by the following modified single profile:

$$U^+ = 2{\cdot}5 \ln(1 + 0{\cdot}4\, Y^+) + 7{\cdot}8\left[1 - \exp\left(-\frac{Y^+}{11}\right) - \left(\frac{Y^+}{11}\right)\cdot \exp(-0{\cdot}33\, Y^+)\right], \quad (Y^+ \geqslant 0). \qquad (5)$$

This velocity profile, originally due to Reichardt [14], has the coordinates modified to U^+ and Y^+ as defined in the Nomenclature and explained in [12] where

$$Y = \frac{r_2^2 - r_m^2}{r_2} - \left[\left(\frac{r_2^2 - r_m^2}{r_2}\right)^2 - r_2^2 + r^2 + 2r_m^2 \ln\left(\frac{r_2}{r}\right)\right]^{0{\cdot}5} \qquad (6)$$

Now, the eddy diffusivity for momentum, ε_M, can be determined from equation (3), knowing the velocity profile given by equation (6).

In fully developed turbulent flow in a concentric annulus, the mean static pressure can be assumed to be constant across any given cross section and the shear stress variation can be obtained from a force balance on an annulus fluid element as

$$\tau(r) = \frac{\tau}{\tau_2} = \left(\frac{r_2}{r}\right)\cdot\left(\frac{r_m^2 - r^2}{r_2^2 - r_m^2}\right). \qquad (7)$$

The following relation can be written for equation (3) from equation (5),

$$\frac{\mathrm{d}U^+}{\mathrm{d}r^+} = \frac{\mathrm{d}U^+}{\mathrm{d}Y^+}\frac{\mathrm{d}Y^+}{\mathrm{d}r^+} = \phi_1(Y^+)\,\phi_2(r^+) \qquad (8)$$

where

$$\phi_1(Y^+) = \frac{1}{1 + 0{\cdot}4Y^+} + \frac{7{\cdot}8}{11}\left[\exp\left(-\frac{Y^+}{11}\right) - \exp(-0{\cdot}33Y^+) + 0{\cdot}33Y^+\exp(-0{\cdot}33Y^+)\right] \qquad (9)$$

and

$$\phi_2(r^+) = -\frac{\left(\dfrac{r^{+2} - r_m^{+2}}{r^+}\right)}{\left[\left(\dfrac{r_2^{+2} - r_m^{+2}}{r^{+2}}\right)^2 - r_2^{+2} + r^{+2} + 2r_m^{+2}\ln\left(\dfrac{r_2^+}{r^+}\right)\right]^{0{\cdot}5}}. \qquad (10)$$

Therefore, from equations (3, 7, 8), the following equation is obtained.

$$(\varepsilon_M/\nu) = [\tau(r)/\phi_1(Y^+).\ \phi_2(r^+)] - 1. \quad (11)$$

The eddy diffusivity of momentum calculated from equation (11) gives the maximum values of ε_M at the point of maximum velocity. As pointed out in [1, 7], there is evidence that ε_M does not go to a maximum value, nor to zero at the point of maximum velocity. For this study, two cases were considered in the case of fully developed flow:

Case (i). The value of ε_M is permitted to increase to maximum at the point of maximum velocity (zero shear).

Case (ii). The value of ε_M was permitted to increase up to the point one third of the passage and kept constant over the center third.

The link between equations (3) and (4) is the ratio, $\sigma(r)$. The ratio $\sigma(r)$ is a function not only of Reynolds and Prandtl numbers, but also of the position in the cross section of the annular space. With air, $\sigma(r)$ seems to decrease from a value of 1·4 in the sub-layer region towards unity outside the so-called buffer layer, in much the same way as that measured in pipe flow [7]. Therefore in the present analysis for the case of Prandtl number of 0·72, it was first decided that although a single modified velocity profile was used, the velocity profile was divided into two regions in the determination of temperature distribution to allow for the variation of $\sigma(r)$ in the neighbourhood of the heated core wall as follows: (a) a wall region in the inner part of the annular space in order to study the effect of a (b) the remaining part. In the region (a) near the inner wall $\sigma(r)$ was taken to be $\sigma(r) = 1{\cdot}4 - (Y^+/75)$ and in the remaining region, referred to as region (b), $\sigma(r)$ was taken to be constant—unity. The results for other ranges of Prandtl numbers were calculated on the assumption that $\sigma(r) = 1$ throughout the annular space.

These assumptions were also taken to be valid in the thermal entrance region. Abbrechet and Churchill [15] from their experimental work on pipe, concluded that the eddy diffusivity for heat was independent of length in the thermal entrance region and hence a function only of fluid motion, although the work of Hanratty [16] indicates that at smaller values of x the values of eddy diffusivity for heat near the center of the pipe are seen to decrease.

However, for the asymmetric case as in the present study, this difference in the eddy diffusivity obtained by the two workers from their experiments is of little importance, since the heat flux is small in the outer region. Therefore, it is assumed that the $\sigma(r)$ variation in the thermal entrance region of the concentric annulus is independent of the length, x.

Now, equation (4) can be rearranged, by introducing non-dimensional parameters, T^+, Pr and r^+ as

$$\frac{q}{q_1} = \left[\frac{1}{Pr} + \sigma(r)\frac{\varepsilon_M}{\nu}\right]\frac{\mathrm{d}T^+}{\mathrm{d}r^+}. \quad (12)$$

To solve equation (12), the variation of heat flux, $q(r)$ must be known. In order to be able to express the local heat flux q in terms of the known wall value, q_1, it is necessary to make some assumptions concerning the velocity distribution. This matter has been studied in detail by Barrow [17] along with a discussion on eddy diffusivity in the fully developed flow in an annulus. To express the local heat flux q in terms of q_1 in a concentric annulus, he obtained the following equation:

$$q = q_1\frac{r_1}{r}\left(\frac{r_2^2 - r^2}{r_2^2 - r_1^2}\right) \quad (13)$$

assuming that the local velocity is constant and equal to the bulk velocity. He then applied a seventh power law velocity profile across the annular space in order to study the effect of a more realistic velocity distribution and the resulting expression was compared with equation (13). The comparison showed that the more realistic velocity distribution predicted heat fluxes in the outer region of the maximum velocity of the annulus smaller than those calculated from equation (13). However, since the heat fluxes are already very small in the outer region of the annulus, the assumption concerning the velocity field is not critical to the evaluation of $q(r)$ in the region of greatest importance. Therefore, equation (13) was modified in the entrance region as

$$q(r^+)_x = \left(\frac{q}{q_1}\right)_x = \frac{r_1^+}{r^+}\left(\frac{r_h^{+2} - r^{+2}}{r_h^{+2} - r_1^{+2}}\right). \quad (14)$$

According to Deissler and Eian [18], the linear variation of heat flux and shear stress across a pipe gives very nearly the same temperature and velocity distributions as those obtained with uniform heat flux and shear stress. The assumed distribution of heat flux given is therefore not vital to the analysis but the variation given by equation (14) is close to that in the real situation and is therefore preferred.

With $q(r)$ known, the combination of equations (11, 12, 14) yields upon integration the following expression for the temperature

$$T^+ = \int_{r_1^+}^{r^+} \frac{q(r^+)_x \, dr^+}{(1/Pr) + \sigma(r^+)\{[\tau(r^+)/\phi_1(Y^+).\phi_2(r^+)] - 1\}}. \quad (15)$$

2.3. *Nusselt number and Reynolds number*

The local heat-transfer coefficient and Nusselt number are defined in the usual way as

$$h_x = \frac{q_1}{(T_1 - T_b)_x} \quad (16)$$

$$Nu_x = \frac{h_x De}{k} \quad (17)$$

and Reynolds number as

$$Re = \frac{\rho u_b De}{\mu}. \quad (18)$$

Now, the bulk temperature, T_b, and bulk velocity, u_b can be defined, in terms of the dimensionless variables, as

$$(T_b^+)_x = \frac{\int_{r_1^+}^{r_2^+} T^+ U^+ r^+ \, dr^+}{\int_{r_1^+}^{r_2^+} U^+ r^+ \, dr^+} \quad (19)$$

and

$$U_b^+ = \frac{2\int_{r_1^+}^{r_2^+} U^+ r^+ \, dr^+}{(r_2^{+2} - r_1^{+2})}. \quad (20)$$

Therefore, the expressions of the Nusselt number and Reynolds number for concentric annuli can be written as

$$Nu_x = \frac{2r_1^+(\alpha - 1)\, Pr}{(T_b^+)_x} \quad (21)$$

and

$$Re = \frac{4\int_{r_1^+}^{r_2^+} U^+ r^+ \, dr^+}{r_1^+(\alpha + 1)}. \quad (22)$$

The equation (21) can be used in the entrance region as well as for the fully developed case. However, for $(r^+ - r_1^+)/(r_2^+ - r_1^+) \geqslant \delta_h^+$, it is necessary that $T^+ = T_h^+$.

2.4. *Computational procedures*

The numerical solutions in this analysis were obtained with the KDF-9 digital computer at the Computer Laboratory, University of Liverpool. The machine language used was LU/ALGOL.

In the computing process, the parameter r_1^+ is used as the independent variable with radius ratio, α. A graph of the relationship between r_1^+ and Reynolds number is made for various radius ratios prior to the actual computation. This was obtained from equation (22) and gives the appropriate value of r_1^+ to be used with respect to a given radius ratio when the Nusselt numbers and the value of x/De for a corresponding Reynolds number are to be calculated, the values of δ_h^+ being fixed.

Calculations were made for Reynolds numbers ranging from 10000 to 200000, for radius ratios from 1·01 to 5 and for Prandtl numbers from 0·01 to 30.

3. RESULTS AND DISCUSSION

The development of the non-dimensional temperature profiles in the thermal entrance region of a concentric annulus for uniform heat flux at the core and a fully developed velocity profile calculated from equation (15) are presented in Fig. 2.

Fully developed temperature distributions at a radius ratio of 1·5 and Reynolds number 10000 are plotted in Fig. 3 for Prandtl numbers 0·01, 0·1, 1, 3 and 5. The effect of the eddy diffusivity on the temperature at various values of Prandtl number is clearly seen. At high Prandtl numbers, the temperature gradient close to the core wall is extremely large, in which case the molecular heat transfer term can be neglected in the region away from the core wall.

A representative comparison of the predicted temperature distribution with the experimental results of [7] is shown in Fig. 4 as (T^+/T_h^+) against the normalised wall distance. In this way, the use of wall heat flux, q_1, can be avoided. The agreement between experiment and the present analysis is excellent in the case of air as the working fluid, $Pr \doteq 0{\cdot}72$.

The numerical values in the present analysis calculated with different eddy diffusivity distribution mentioned in the Section 2·2 showed very little difference. For example, fully developed Nusselt numbers calculated with the distribution of eddy diffusivity for case (i) was 113·42 while for case (ii) it was 113·14 when the Reynolds number was 50000, Prandtl number 0·72 and

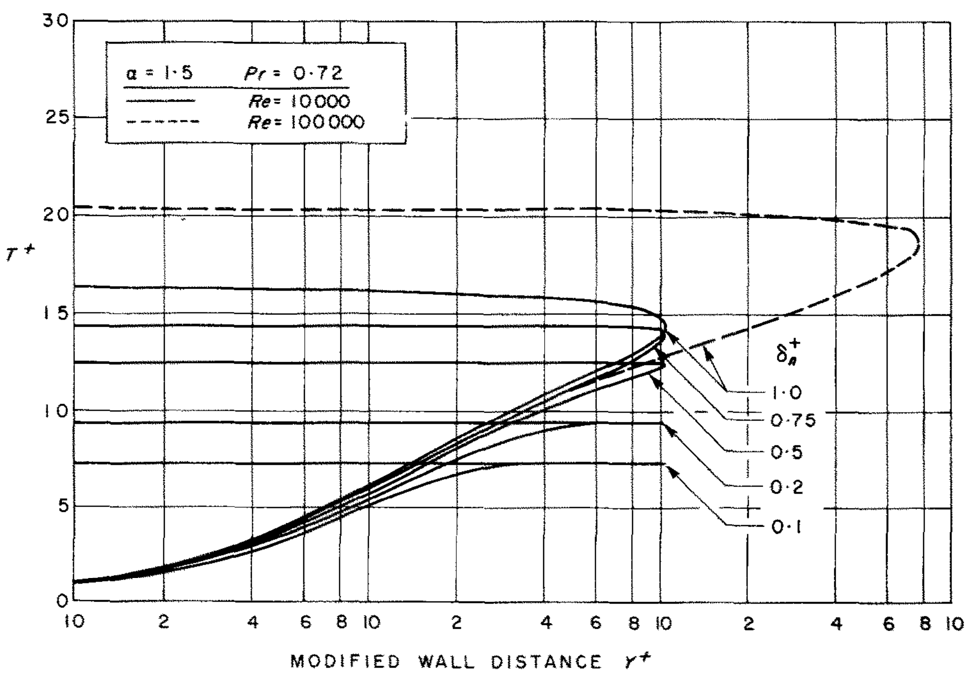

FIG. 2. Predicted temperature distribution.

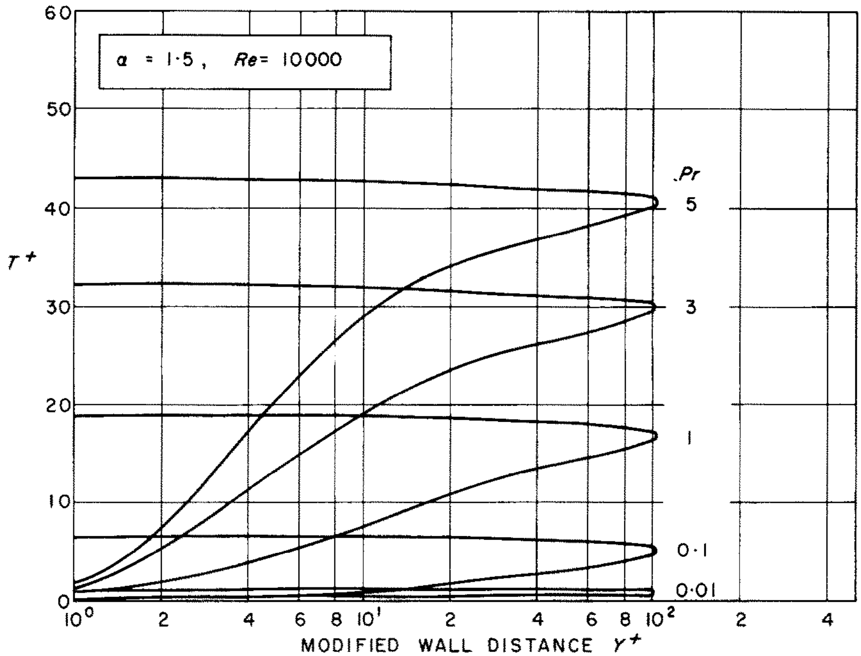

FIG. 3. Fully developed temperature profiles.

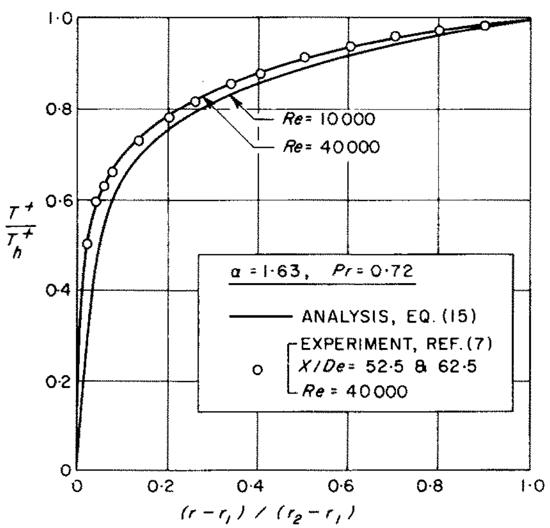

FIG. 4. Comparison of fully developed temperature profile with experiment.

the radius ratio 1·5. Therefore all the results in the present analysis were calculated with the distribution of eddy diffusivity of case (i).

The curves in Figs. 5 (a–c) show the effect of Reynolds number on the values of the ratios (Nu_x/Nu_d) in the thermal entrance region of an annulus at a given value of x/De and Prandtl number. It is seen that for a given Prandtl number and α, Reynolds number seems to have minor influence on entrance effects. The values of (Nu_x/Nu_d) very near to the entrance are lower for the high Reynolds numbers than for the low ones, for a given Prandtl number.

To illustrate the effect of Prandtl number, a comparison is made in Fig. 6 as a function of x/De for three values of Prandtl number of 0·1, 1 and 10. For a fixed Reynolds number and radius ratio, the entrance effects at a given distance, x/De, decrease with increasing Prandtl number, the same trends as observed in the pipe flow case [4, 5]. Observation of the results in Fig. 6 shows that Nusselt numbers nearly reach their fully developed values long before the thermal boundary layers are fully developed. For example, $\alpha = 1{\cdot}5$, at Reynolds number

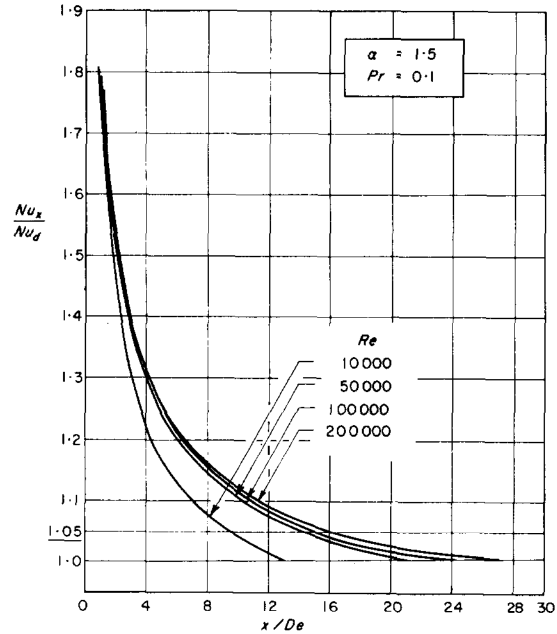

FIG. 5(a). Entrance region Nusselt numbers.

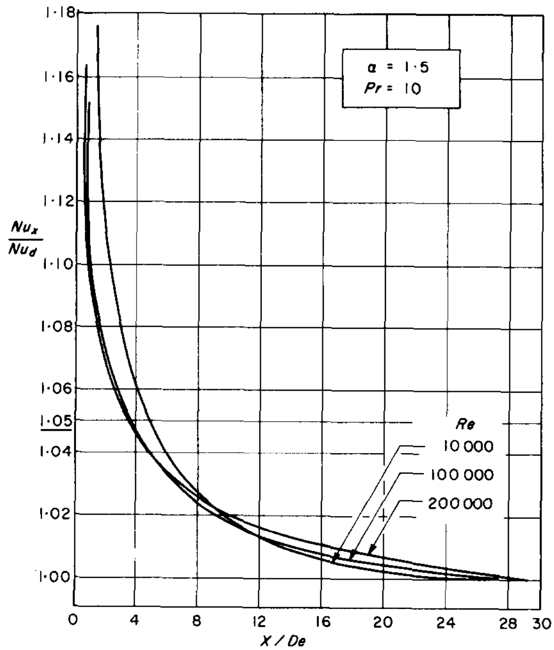

Fig. 5(c). Entrance region Nusselt numbers.

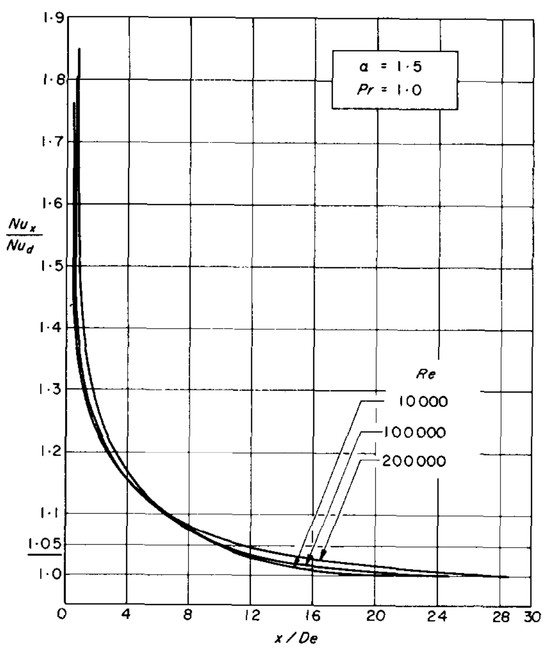

FIG. 5(b). Entrance region Nusselt numbers.

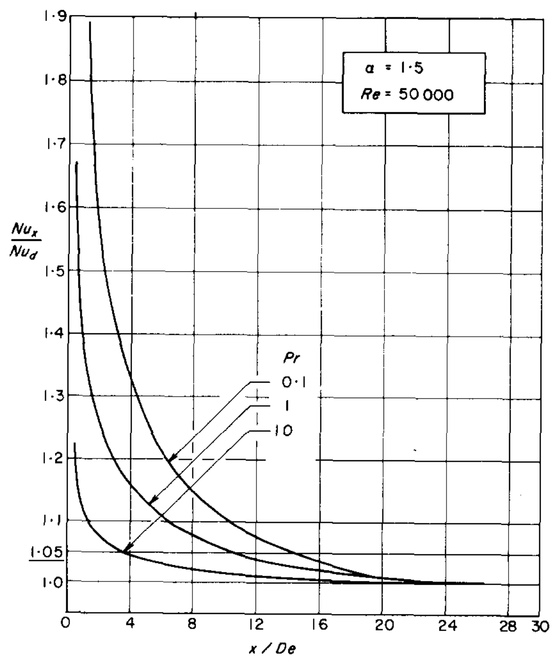

FIG. 6. Entrance region Nusselt numbers.

= 50000 and Prandtl number = 1·0, the Nusselt number at $(x/De)_x \doteq 10$ is within 5 per cent of its fully developed value at $(x/De)_d = 21{\cdot}35$.

Sparrow *et al.* [5] state in their analysis of the thermal entrance region in pipe flow that there is a marked decrease in the entrance length as the Prandtl number increases and that this is substantiated by the results of Deissler's analysis [4] and of experiments by Boelter *et al.* [19], and Hartnett [6]. However, Hartnett concluded from his experiment work [6] that the entrance length, $(x/De)_d$, is unaffected by Prandtl number when Prandtl number is greater than one. He reported later that some effect of Prandtl number was noticed (see Discussion in [6]).

Attention must now be brought to the definition of the thermal entrance length used by the different investigators. The definition used by Sparrow *et al.* [5] is based on $(Nu_x/Nu_d) = 1{\cdot}05$; i.e. that distance required for the local heat-transfer coefficient to approach to within 5 per cent of the fully developed value. The values of $(x/De)_d$ presented in the present work are those based on $(Nu_x/Nu_d) = 1{,}00$; i.e. that distance from the entrance to the cross section where temperature profile has no straight-line portion normal to the fluid flow axis.

Therefore, in the present analysis the values of $(x/De)_d$ corresponding to $(Nu_x/Nu_d) = 1{\cdot}00$ are plotted in Fig. 7 for three Prandtl numbers along with the values of $(x/De)_d$ corresponding to $(Nu_x/Nu_d) = 1{\cdot}05$ which have been read from Figs. 5(a–c).

It is of great interest to note from Fig. 7 that the effect of Prandtl number on $(x/De)_d$ is in completely opposite agreement according to the definition used.

By definition based on $(Nu_x/Nu_d) = 1{\cdot}00$, the thermal entrance length, $(x/De)_d$, increase as the Prandtl number increase.

However, if the definition of thermal entrance length based on $(Nu_x/Nu_d) = 1{\cdot}05$ is taken, the values of $(x/De)_d$ decrease markedly with increasing Prandtl number which is in accord with the findings of Sparrow *et al.* [5]. The effect of the definition of $(x/De)_d$ is more critical with high Prandtl number. For example, at a Reynolds number of 100000 for a radius ratio of 1·5, for Prandtl number of 10, $(x/De)_d$ based on $(Nu_x/Nu_d) = 1{\cdot}05$ is about 3·5, which is less that 15 per cent of the value for fully developed flow.

To distinguish between heat-transfer coefficients differing from each other by only 5 per cent, is practically impossible in an experimental investigation, since the spread of experimental data in the heat-transfer field is usually greater than 5 per cent, no matter how careful and complete the experimental programme may be.

Therefore, it may be concluded from the above discussion that the effect of Prandtl number on the thermal entrance length, $(x/De)_d$, is extremely difficult to determine experimentally.

The effect of radius ratio on the variation of Nusselt numbers and entrance lengths can be seen in Fig. 8. The values of (Nu_x/Nu_d) for smaller α are higher than those for a larger α. At this point, it must be mentioned that the results of Kays and Leung [1] should be comparable but their presentation of results were such that direct comparison with the present work was not possible. The values of the fully developed thermal entrance lengths, $(x/De)_d$, based on $(Nu_x/Nu_d) = 1{\cdot}00$ are also plotted in Fig. 9 for various values of radius ratios. The comparison indicates that the values of $(x/De)_d$ for smaller α are again larger than those for larger α.

Predicted Nusselt numbers for fully developed heat transfer, calculated from equation (21) are plotted against Reynolds number for various values of Prandtl number in Fig. 10(a) and for various values of radius ratio in Fig. 10(b). In Fig. 10(b), the pipe flow case (e.g. Deissler work [4]) has not been compared with the results of the present analysis as the heating boundary conditions are different; however, the case of heat transfer from one wall of a parallel plate channel [20] has been compared, the heating boundary conditions being compatible. The curve for this case by the present analysis, that is for $\alpha \doteq 1$, agrees very closely with that given in [20]. The increasing Nusselt number with increasing radius ratio can be also seen.

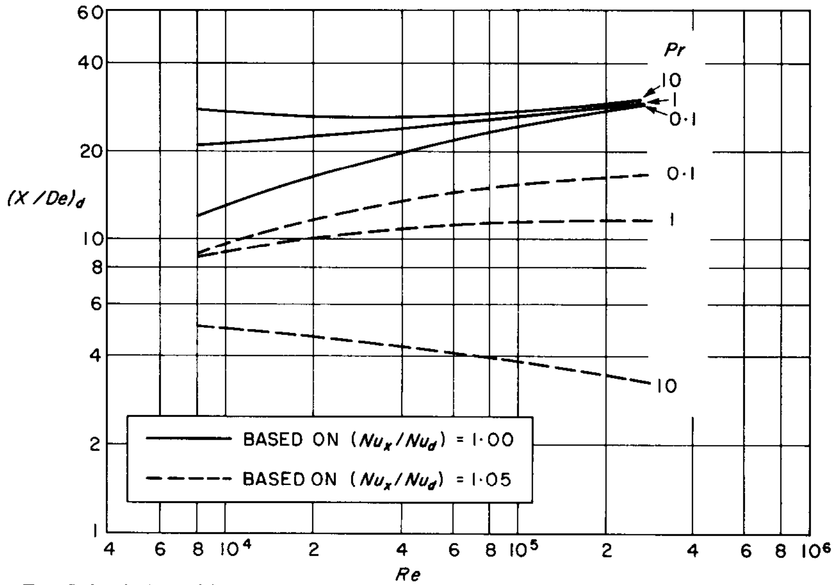

FIG. 7. Variation of fully developed thermal entry lengths with Re and Pr, $\alpha = 1·5$.

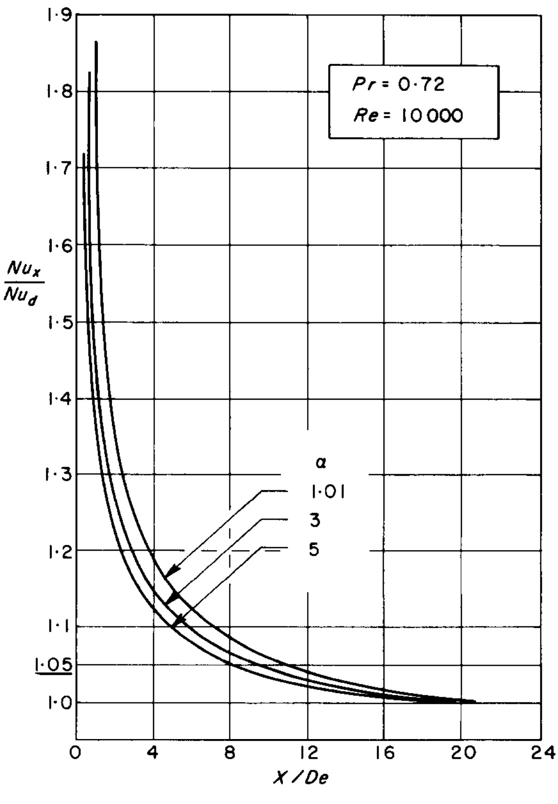

FIG. 8. Entrance region Nusselt numbers.

Comparison between the present theory and the results of other workers for the fully developed heat transfer in an annulus is made in Fig. 11, in which Nusselt numbers are plotted against Reynolds number for a radius ratio of 3·875. The experimental Nusselt numbers and Reynolds numbers of [7] were obtained at points $(x/De) = 52·5$ and $62·5$ from the entrance where the flow was practically fully developed and the physical properties were evaluated at the bulk temperature.

Despite the different method of analysis, the results of Barrow [17] are in good agreement with that of present analysis in the range of Reynolds numbers shown, though the slope of Barrow's curve is slightly lower than that of the present. However, Barrow also employed an almost identical modified velocity profile in his analysis and this indicates the importance of the assumption regarding the velocity profile for an annular flow heat-transfer analysis.

The difference between the analytical results of Barrow [17] and the present author, and that of Kays and Leung [1] on the fully developed heat transfer is due possibly to differences in the premise on velocity field in an annulus. The curves of Davis [21], Monrad and Pelton [22]

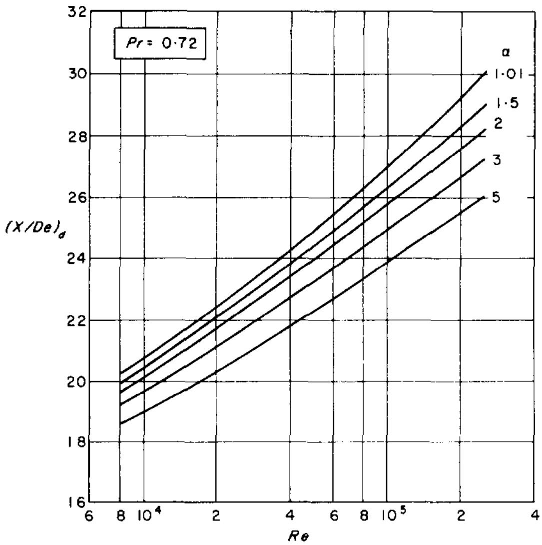

FIG. 9. Fully developed thermal entry lengths.

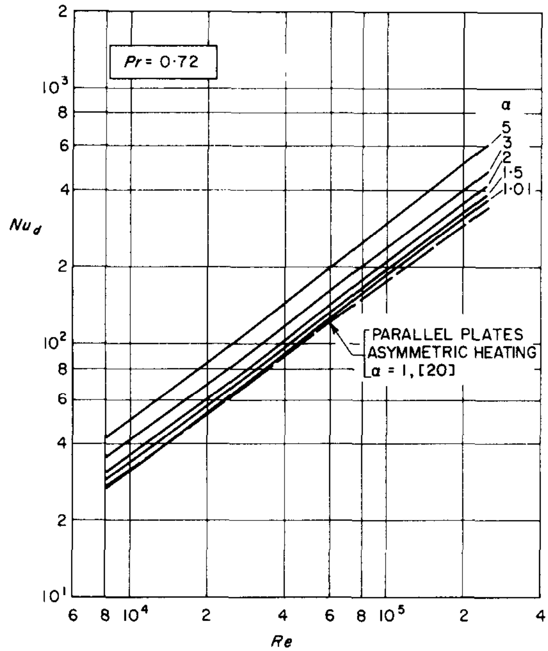

FIG. 10(b). Fully developed Nusselt numbers.

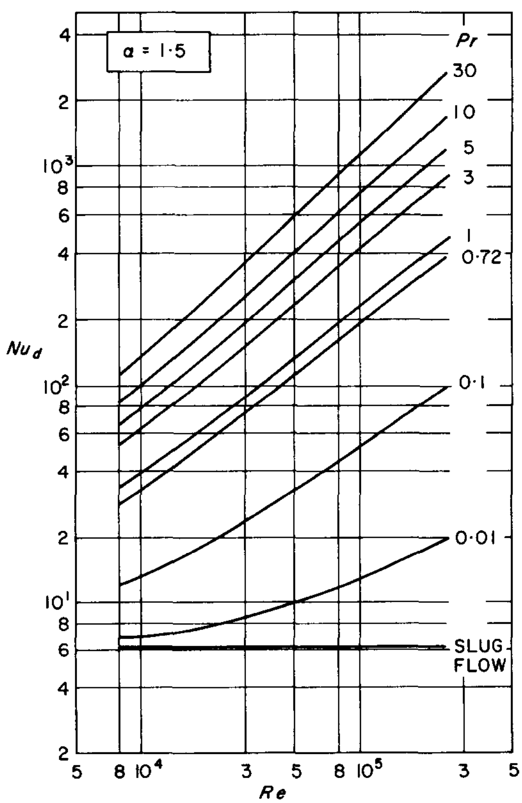

FIG. 10(a). Fully developed Nusselt numbers.

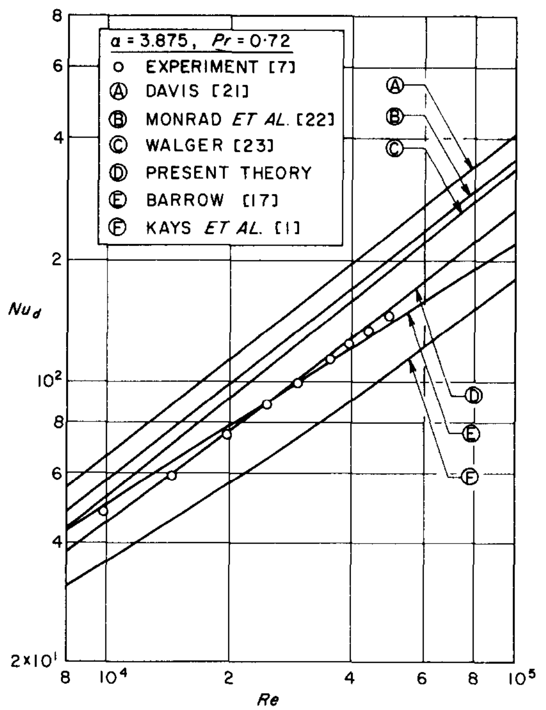

FIG. 11. Comparison of fully developed Nusselt numbers with other investigations.

and Walger [23] are all empirical correlations and are much higher than those of this analysis.

4. CONCLUSIONS

The following conclusions were derived from the analytical investigation of the local heat-transfer characteristics and entrance length for fluid flowing turbulently in concentric annuli with a fully developed velocity profile and a uniform heat flux at the core wall:

1. The fully developed turbulent heat transfer based on $(Nu_x/Nu_d) = 1{\cdot}00$, was attained generally in an entrance length of less than thirty equivalent diameters for the range of fluid flow studied.
2. The effect of increasing the radius ratio was to decrease (Nu_x/Nu_d) for a given x/De and Reynolds number, and also increase in the radius ratio leads to decrease in the thermal entrance length, but to increase in the Nusselt number.
3. For low Prandtl number, increase in Reynolds number leads to an increase in $(x/De)_d$ based In $(Nu_x/Nu_d) = 1{\cdot}00$; at very high values of Prandtl number, the variation of $(x/De)_d$ with Reynolds number is more complex. However, the effect of Reynolds number has, in general, a minor influence on $(x/De)_d$ for Prandtl number greater than unity.

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Résumé—On étudie les problèmes de la croissance de la couche limite thermique et du transport de chaleur, pour un écoulement turbulent entièrement établi au point de vue hydrodynamique dans un tuyau annulaire à cylindres concentriques au moyen des équations intégrales de la quantité de mouvement et du transport de chaleur avec un profil universel modifié de vitesse.

La recherche était conduite dans une gamme de rapport de rayons de 1,01 à 5, de nombres de Prandtl de 0,01 à 30 et de nombres de Reynolds de 10000 à 20000.

Les résultats révèlent qu'en général le coefficient de transport de chaleur atteint la valeur du régime entièrement établi en moins de 30 diamètres équivalents; et en outre que la longueur d'entrée dépend d'une façon modérée du rapport des rayons.

Il y a un bon accord entre l'analyse actuelle et les données expérimentales existentes pour les régions du profil de température entièrement établi.

Zusammenfassung—Probleme des Anwachsens der thermischen Grenzschicht und des Wärmeübergangs bei hydrodynamisch voll ausgebildeter turbulenter Strömung in konzentrischen Ringräumen werden mit Hilfe der Bewegungs- und Wärmeübergangsgleichung unter Verwendung eines modifizierten universellen Geschwindigkeitsprofiles gelöst.

Die Untersuchung wurde durchgeführt für Radiusverhältnisse von 1,01 bis 5, Prandtl-Zahlen von 0,01bis 30 und einem Reynolds-Zahlenbereich von 10000 bis 200000.

Die Ergebnisse zeigen, dass im allgemeinen der Wärmeübergangskoeffizient seinen Endwert nach einer Länge von weniger als dreissig hydraulischen Durchmessern erreicht, und dass die Einlauflänge etwas vom Radiusverhältnis abhängt.

Es besteht gute Übereinstimmung zwischen der gegenwärtigen Analyse und bekannten Versuchswerten für voll ausgebildete Temperaturprofilbereiche.

Аннотация—Задачи роста теплового пограничного слоя и теплообмена для полностью развитого турбулентного потока в кольцевых каналах исследовались с помощью интегральных уравнений теплопереноса и количества движения, а также с помощью унифицированного универсального профиля скорости.

Исследование проводилось в диапазоне отношений радиусов от 1,01 до 5, чисел Прандтля от 0,01 до 30 и чисел Рейнольдса от 10000 до 20000.

Результаты показывают, что, в общем, коэффициент теплообмена достигает величины для полностью развитого температурного профиля не менее, чем через 30 эквивалентных диаметров, а также, что длина начального участка в некоротой степени зависит от отношения радиусов.

Существует хорошее согласование между данным теоретическим анализом и имеющимися экспериментальными данными для областей полностью развитого температурного профиля.